Calibration of the broad band UV Radiometer

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ABSTRACT

Mounting concern about the ozone layer depletion and the potential ultraviolet exposure increase accelerate the need for an accurate ultraviolet radiation monitoring. To assure the accuracy of measurement the instrument has to be well characterized and the calibration procedure has to be designed to meet the growing precision and accuracy requirements. In depth error analysis is also necessary to properly estimate the character and amplitude of errors incurred during the calibration procedure. A method for calibration of a broad band UV radiometer is proposed. In order to achieve the highest precision it is based on spectroradiometric transfer from a standard lamp and a standard detector in a well controlled laboratory conditions. Random and systematic error sources are identified and their contribution to the final calibration result is calculated. The sensitivity of the calibration to the measurement errors and the errors of the references varies with wavelength. Statistical dependency of measurements within a spectroradiometric scan contributes significantly to the estimate of calibration precision. The calibration formula is linearly approximated with the first component of its Taylor series and the variance of the calibration is approximated based on the covariance matrices of the error sources. The analytical and numerical methods of error estimation presented can be applied to a broad range of radiometers and spectroradiometers

1. Calibration Procedure

1.1. Assumptions

The following assumptions are made during the calibration

- meters are fully characterized both spectrally and in terms of angular response at nominal sensor temperature
- calibration is corrected for a standard sun defined as the output of the UV radiation model[1] under 2.7mm ozone column 30° solar zenith angle (SZA), at sea level and zero albedo
- The meter is calibrated in MED/Hr (Minimum Erythema Dose per Hour). The conversion factor *M* between Erythemally [2] weighted power and MED/Hr is[3] : M=17.1 [(MED/Hr)/(W/m^2)]
- calculations and measurement are limited to a 270 - 400nm range
- stable 150W Xe arc lamp with 1mm WG305 filter is used as a calibration source; the source is measured before each calibration and measurement is repeated at least every hour

1.2. Principle of calibration.

The current *I* generated by the detector with the absolute spectral response $R^d(\lambda)$ under incident spectral irradiance $E(\lambda)$ is:

$$I = \sum_{\lambda} R^d(\lambda) \cdot E(\lambda) \cdot \Delta\lambda \quad [\text{Amps}] \tag{1}$$

With a known calibration factor *K* [(MED/Hr)/Amp] the meter reads:

$$S = K \cdot I = K \sum_{\lambda} R^d(\lambda) \cdot E(\lambda) \cdot \Delta\lambda \qquad [\text{MED / Hr}] \tag{2}$$

During the calibration procedure the meter's calibration factor *K* is adjusted so that in front of the calibration UV source with a measured irradiance $E^{Xe}(l)$ the meter reads:

$$S^{Xe} = \frac{\sum_{\lambda} r^{d}(\lambda)\cdot E^{Xe}(\lambda)\cdot\Delta\lambda \sum_{\lambda} M\cdot E^{Sun}(\lambda)\cdot R^{Ery}(\lambda)\cdot\Delta\lambda}{\sum_{\lambda} r^{d}(\lambda)\cdot E^{Sun}(\lambda)\cdot\Delta\lambda} \qquad \textbf{(3)}$$

where E^{Sun} is the spectral irradiance form a standard sun, R^{Ery} is the Erythema Action Spectrum and r^d is a relative spectral response of the detector. The absolute spectral response of the meter R^d is extremely difficult to measure directly so it will be proven that the relative spectral response r^d is sufficient for the purpose of this calibration and that the meter calibrated according to (3) will read accurately the Erythemal Effectiveness of the standard sun.

From (2) the reading in a front of the calibration source can be expressed as:

$$S^{Xe} = K\sum_{\lambda} R^{d}(\lambda)\cdot E^{Xe}(\lambda)\cdot\Delta\lambda \qquad \textbf{(4)}$$

The calibration factor *K* can be derived from (3) and (4):

$$K = \frac{\sum_{\lambda} M\cdot E^{Sun}(\lambda)\cdot R^{Ery}(\lambda)\cdot\Delta\lambda}{\sum_{\lambda} R^{d}(\lambda)\cdot E^{Sun}(\lambda)\cdot\Delta\lambda} \qquad \textbf{(5)}$$

Equation (5) shows, that the result of the calibration procedure does not depend on the spectrum of the calibration source E^{Xe} providing it is accurately measured before each calibration and does not change during the calibration. Based on (2) and (5) the reading of the calibrated detector exposed to the standard sun is:

$$S^{Sun} = M\sum_{\lambda} E^{Sun}(\lambda)\cdot R^{Ery}(\lambda)\cdot\Delta\lambda \qquad \textbf{(6)}$$

which by definition is the erythemal effectiveness of the standard sun. In other words, all calibrated meters should give the same reading under the standard sun no matter what is the spectral response but with the assumption all have the same angular response. Under different solar conditions the measured erythemal effectiveness will have a predicable error that depends on the difference between the R^{Ery} and r^d.

1.3. Instrumentation.

All spectral measurements were performed with double grating spectroradiometer Model 740A/D from Optronic Laboratories. Entrance, middle and exit slits were chosen to provide a 2.5nm half bandwidth. It gave a good compromise between the signal to noise ratio and the systematic measurement error.

A 200W quartz halogen lamp Model 220A supplied and calibrated by Optronic Laboratories and traceable to NIST served as an irradiance standard. The lamp current was stabilized by Model 65DS Constant Current Source with a specified accuracy of 0.1%. It results in a potential 1% error systematic irradiance error around 300nm. For calibration the lamp was positioned at a distance of 50cm.

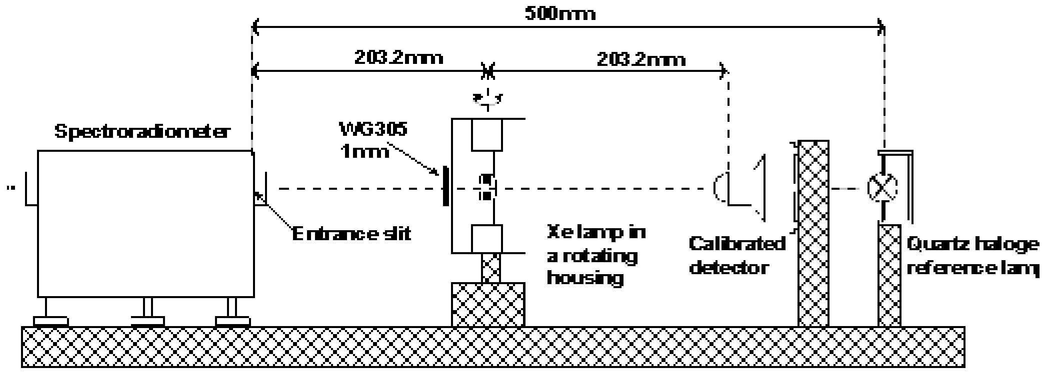

Fig. 1. The calibration setup.

The monochromatic output of the system, needed for measurement of the detector spectral response, was determined with standard silicon photodetector Model 730-5C calibrated and supplied by Optronic Laboratories. The specified 3σ uncertainty in the discussed wavelength range was estimated at 6%.

Super quiet, high pressure, 150W xenon arc lamp type L2274 from Hamamatsu was powered by a very stable xenon lamp Power Supply Model XPS200 manufactured by the Solar Light Company. After an initial warm-up the current regulation is better than 0.2%. A 1mm thick WG305 Schott filter is positioned in the front of the xenon lamp to absorb the very short, unstable UV radiation emitted by the lamp.

2. Error Analysis

2.1. Random calibration errors

When calculating the overall calibration random error which is a result of the measurement random errors as well as standard detector/source errors the following has to be taken into account:

- The standard deviations of the relative measurement error are wavelength dependent (Fig. 2). due to signal/noise ratio change. The shape of the measured spectral distribution has great effect.

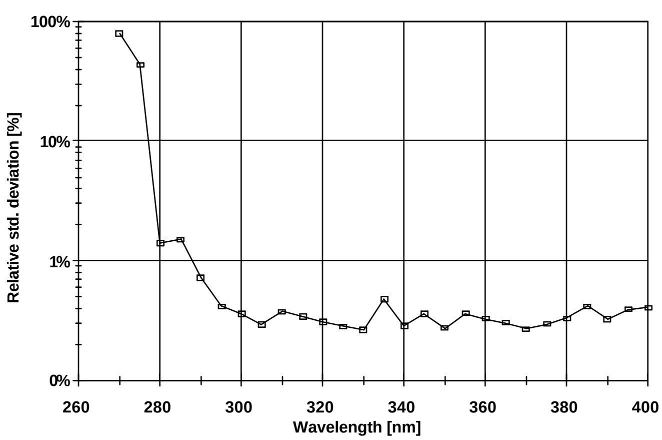

Fig. 2. Standard deviation of the series of spectroradiometric measurements of the calibration source E^{Xe}.

- The error of spectroradiometric measurements at different wavelengths does not form a vector of statistically independent random variables. The covariance matrix (Fig. 3) contains the variances of random variables on the diagonal and the values of 2nd degree central moment of their joint distributions elsewhere.
- Errors at different wavelengths contribute to the overall calibration error with different weight (Fig. 4)

The quantities measured during the calibration process are:

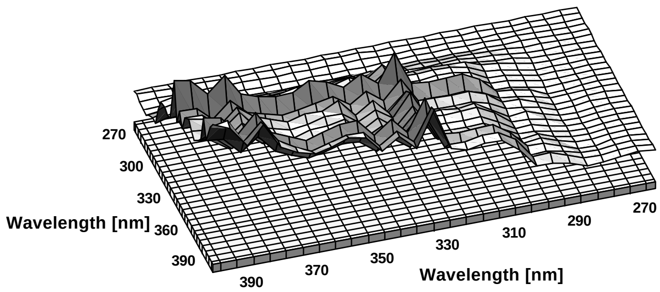

Fig. 1. Covariance matrix of a series of spectroradiometric measurements of I^q. For independent processes $cov(x_i,x_j)=0$ for $i\neq j$, which is not the case here.

I^c -current from the reference detector during the monochromatic output calibration

I^d - current from the measured detector during measurement of its spectral response

I^q - photodetector current during the spectroradiometer calibration

I^{Xe} - photodetector current during the calibration source measurement

In this notation

$$r^d(\lambda) = \frac{I^d(\lambda)\cdot R^c(\lambda)}{I^c(\lambda)} \quad \text{and} \quad E^{Xe}(\lambda) = \frac{I^{Xe}(\lambda)\cdot E^q(\lambda)}{I^q(\lambda)} \tag{1}$$

where $R^c(\lambda)$ is the spectral response of the standard detector and $E^q(\lambda)$ is the spectral irradiance of the standard lamp. The calibration formula (**Error! Bookmark not defined.**) was expanded to include the system calibrations:

$$S^{Xe} = \frac{\sum_\lambda \frac{I^d(\lambda)\cdot R^c(\lambda)}{I^c(\lambda)}\cdot\frac{I^{Xe}(\lambda)\cdot E^q(\lambda)}{I^q(\lambda)}\cdot\Delta\lambda \ \sum_\lambda M\cdot E^{Sun}(\lambda)\cdot R^{Ery}(\lambda)\cdot\Delta\lambda}{\sum_\lambda \frac{I^d(\lambda)\cdot R^c(\lambda)}{I^c(\lambda)}\cdot E^{Sun}(\lambda)\cdot\Delta\lambda} \tag{2}$$

The covariance matrices for all the current measurements were calculated numerically based on a series of measurements. The covariance matrices for the calibrated detectors and sources are not provided. Statistical independence of errors at different wavelengths is therefore assumed. Consequently, a covariance matrix with variance values on the diagonal and zeros elsewhere was generated for both the standard detector and standard lamp. An assumption about normal error distribution was made both for consecutive realizations of the standard as well as for the error distribution with wavelength. The systematic component is lost here but there was no data available to assume otherwise.

It was also assumed that measurements of I^c,I^d,I^q and I^{Xe} are independent of each other so that the individual contributions can be calculated and their variances added.

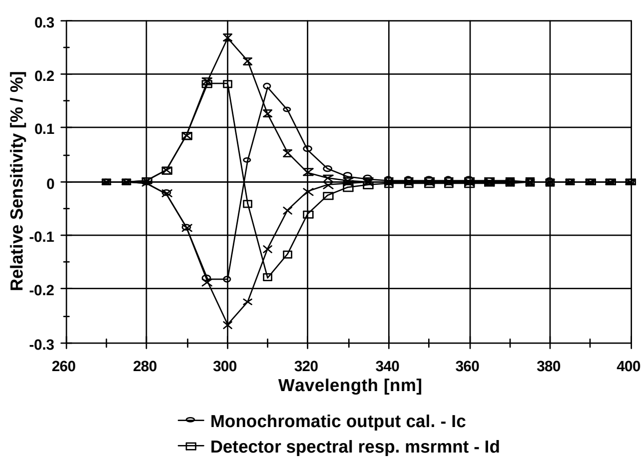

Fig. 2. Sensitivity of the calibration to the measurement errors at various wavelengths.

Analytical evaluation of the S^{Xe} variance would be extremely difficult. the linear approximation approach was chosen and its principle is presented below.

In general a scalar function y=y($\mathbf{x}$) of a vector $\mathbf{x} = [x_1, ... , x_k]^t$ can be represented by its Taylor series around $\mathbf{x}^0$:

$$y(\mathbf{x}) = y(\mathbf{x}^0) + \nabla^t y(\mathbf{x}^0)\Delta\mathbf{x} + \frac{1}{2}\Delta\mathbf{x}^t \ \mathbf{H}(\mathbf{x}^0)\Delta\mathbf{x} + \quad ... \tag{1}$$

where $\mathbf{x}$ is the vector of input variables. The function's continuity and existence of the differentials around $\mathbf{x}^0$ is assumed. By definition, $\Delta\mathbf{x} = \mathbf{x} - \mathbf{x}^0$, the gradient vector is defined as:

$$\nabla y(\mathbf{x}^0) = \left[\frac{\partial\, y(\mathbf{x}^0)}{\partial\, x_1}, \quad ... \quad , \frac{\partial\, y(\mathbf{x}^0)}{\partial\, x_k}\right]^t \tag{2}$$

and $\mathbf{H}(\mathbf{x}^0)$ is a $k \times k$ matrix of second degree differentials (Hessian):

$$H_{ij}(\mathbf{x}^0) = \left[\frac{\partial^{\,2} y(\mathbf{x}^0)}{\partial\, x_i \partial\, x_j}\right] \qquad i, j = 1...k \tag{3}$$

In close proximity to $\mathbf{x}^0$ the first two components of the Taylor series (1) are a good approximation of y($\mathbf{x}$). Introducing random variable $\mathbf{X}$ such that $\mathbf{x}^0$ = E($\mathbf{X}$) the expected value of y is:

$$E[y(\mathbf{X})] \approx E[y(\mathbf{x}^0)] = y(\mathbf{x}^0) \tag{4}$$

because $E[\nabla^t y(\mathbf{x}^0)\Delta\mathbf{X}]=0$. The variance of y can be approximated as:

$$\text{var}[y(\mathbf{X})] = E[y(\mathbf{X}) - y(\mathbf{x}^0)]^2 \approx E[\nabla^t y(\mathbf{x}^0)\cdot\Delta\mathbf{X}]^2 = \nabla^t y(\mathbf{x}^0)\,\mathbf{K}^x\ \nabla y(\mathbf{x}^0) \tag{5}$$

where $\mathbf{K}^X$ is a covariance matrix of a random variable $\mathbf{X}$.

The gradient vector was calculated numerically at the average point of all measured quantities. The random error balance is shown in Table **Error! Bookmark not defined.**.

0.1. Systematic calibration errors

Systematic calibration errors are mainly due to the finite bandwidth and stray light effects on the spectroradiometer measurement. Other major sources of systematic error are: non-linearity of the radiometer, setup error and error of the standard lamp current The systematic components of the standard source and standard detector are not specified. The effect of the slit function exhibits itself most prominently when measuring rapidly changing spectral characteristics, such as the detector's spectral response or the calibration source output. Figure 1 shows the effect of the slit function on the measurement of logarithmically changing spectral irradiance (there is no slit related error on linear slopes if the slit function is symmetrical). The slope was normalized in

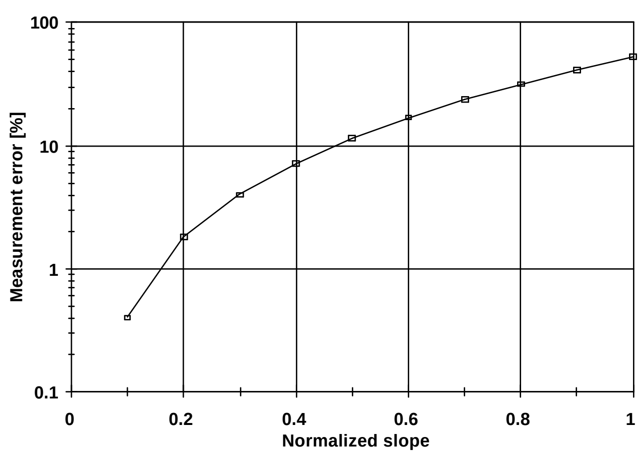

Fig. 1. Measurement error of the logarithmically changing spectral irradiance caused by the slit function. The slope of the spectral irradiance is normalized in relation to half bandwidth of the slit function (decades/FWHM).

relation to the slit's FWHM (decades/FWHM).

Table **Error! Bookmark not defined.** summarizes the systematic error budget in terms of worst case error. The systematic error components of the standard quartz halogen lamp and the standard detector were not separately specified so it was not possible to separate their effects from the random error.

1. Conclusions and Recommendations

- A calibration of the broad band UV radiometer can be performed in relatively simple setup with a random error of 1.6% (1σ) and a systematic error of -6.2 ... +4.7% worst case; this precision is sufficient to detect trends in the order of 10%/decade with 2 calibrations per year
- In-depth error analysis helps to identify the biggest error contributors - spectroradiometer calibration and standard detector uncertainty for the random error component
- It is important to note that the above uncertainty analysis relates to the calibration process only. It means, that if an absoluteley stable detector was repeatedly calibrated with the above method the resulting calibration factor series would have 1.6% standard deviation and under the reference conditions (point source, standard solar spectrum) the average would be within -6.2 to +4.6% of the accurate erythemal reading. In the field additional error sources contribute to the overal measurement uncertainty, such as: spectral difference from erythema, cosine error, pollution of the optics and drifts.
- The estimated error contribution of the standard lamp uncertainty (0.2%) is lower than the uncertainty of the lamp's output in the UV-B region as a result of an assumption about statistical independence of the calibration values. Quite possibly this value is underestimated. However, to calculate it properly we would need a more comprehensive characterization of the lamp uncertainty, namely the covariance matrix, instead just one number typically available from the calibration laboratory. A conservative estimate can be obtained by assuming that this contribution is equal to the standard lamp uncertainty at the peek wavelength of the calibrated detector (approx. 0.6% 1σ in this case). It should be pointed out that the lamp contribution will have a random character for consecutive calibrations only if a new lamp is obtained for each calibration.
- The assumption about statistical independence within the data set may cause either under- or over-estimation of the resulting error, depending on the gradient vector. The contribution of the standard detector is overestimated due to the assumed independence. Whether this assumption will cause over- or under- estimated depends on the shape of sensitivity functions (Fig. 4).
- All spectral measurements and data should be checked for statistical independence within the scan; if data are dependent then a covariance matrix should be used in final error determination; using interpolation algorithms for spectral data introduces dependence to the data set. Excessive dependence may also indicate a problem with the equipment.
- Performing the calibration under well controlled laboratory conditions assures high precision of the calibration, crucial from the point of view of long term stability, at an expense of the absolute accuracy. This calibration method should accompany the typically used transfer from collocated spectroradiometer. It will provide information useful for analysis of calibration coefficients gathered over the years. It is particularly important in situation where the uncertainty of absolute calibration is greater than the meters stability [Error! Bookmark not defined.], which seem to be the case with the R-B meters.
- Great care should be exercised when classifying the uncertainty components. The nature of the particular component should be the criteria, since it determines the way it propagates, and at the same time the mathematical methods used to estimate it. Combining the random, systematic and drift component together rarely simplifies the process, often leading to unreasonable conclusions.

Table 1. Calibration procedure uncertainty - random component summary

Uncertainty source	Comment	Uncertainty contribution (1σ)
Standard lamp uncertainty **Error! Bookmark not defined.**	Provided by calibration lab.	0.2%[1]
Standard detector uncertainty **Error! Bookmark not defined.**	Provided by calibration lab.	0.7%[2]
System calibration error	Includes short term lamp current variations, lamp instability, mechanical setup error and spectroradiometric measurement error	1.3%
Calibration source E^{Xe} measurement	Includes Xe lamp and filter instability, setup error and spectroradiometric measurement error	0.33%
Monochromatic output calibration		<0.1%
Phosphor measurement error		0.3%
Sensor temperature error	The temperature coefficient of the sensor under the calibration source is 0.2%/°C and constant temperature distribution within ±1°C was assumed	<0.1%
Detector adjustment	Includes the calibration source variations, detector/reading device noise, mechanical setup error	0.4%
Total (quadrature sum)		**1.61%**

[1] Underestimated due to an assumption about statistical independence of calibration errors within the wavelength range. See conclusions.

[2] Overestimated due to the same reason as above.

Table 1. Calibration procedure uncertainty - systematic component budget

Uncertainty source	Comment	Uncertainty contribution (worst case %)
Spectroradiometer non-linearity	As specified by manufacturer	±1%
Standard lamp current error	As specified by manufacturer of the power supply	±1%
Detector spectral response measurement	Simulated effect of 2.5nm slit width.	-2 ... 0%
Calibration source measurement (slit function)	Simulated effect of 2.5nm slit width.	0 ... +0.5%
Mechanical setup error (combined for system calibration and calibration source measurement)	Based on estimated uncertainties of the distances between the sources, spectroradiometer and detector	±2%
Sensor temperature systematic error	Assuming 0.2%/°C temp. coefficient for the spectrum of calibration source.	±0.2%
Total (worst case)		**-6.2 ... +4.7%**

References

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